

Postural Equilibrium criteria concerning Feet Properties for Biped Robots

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Research Article

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Abstract:

A research on the postural equilibrium conditions for biped robots is presented in this article. Dynamic walking criteria like ZMP and CoP are presented, and their parallels are examined. The consequences of a compliant foot are also introduced and taken into account while evaluating the criterion. The VICON motion capture technology records a human subject's movements, which are then replicated using a model of a planar biped. Ground reaction forces and body segment accelerations are required to estimate the criterion. For the motion of the model, ZMP and CoP are examined in both single and double support phases. During the double support phase, the biped's load is transferred between the supporting legs using a linear shift function. We compare the simulated CoP trajectories produced by a compliant, deformable spring-damper system situated between the foot sole and the ankle joint, and a rigid foot. It is evident that the deformation of the foot enhances the stability of the biped and smoothes the CoP trajectory.

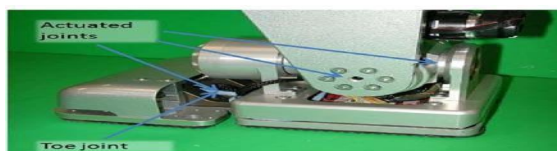
1. Introduction

The role of the ankle and toe joints during bipedal movement has long been a topic of debate. It has been shown that they may store energy upon heel contact and release it during push-off in a manner akin to a spring [2, 4, 10]. This energy release lowers the total energy expenditure of the gait cycle [2]. Naturally, there is a desire to replicate this advantage in walking robots and prosthetic devices [1, 2, 6, 11]. There are two main approaches. The first actively reproduces the motion of the ankle and sometimes the toes. In the second, energy created during heel strikes is stored in a number of springs and released at the proper time 2. This second, compliant approach may reduce the foot's total weight. It may also closely mirror human gait in terms of ground response force distribution at important gait cycle moments [2, 6]. Walking on two feet and calculating the equilibrium criterion The material presented here focuses on ZMP and CoP. These criteria are widely used in the field of walking robots [3, 14, 17]. The next sections provide a brief description of the results obtained and the methodology used to assess these criteria.

Robotic Feet

There are several applications for studying feet in humanoid robots. To expand their range of motion, the humanoid robots H6 and H7 were equipped with an actuated toe-jointed foot in 2002 (see Fig. 1a) [11]. Because to its articulated feet, H6 was able to move more quickly, climb higher stairs, and kneel while keeping the soles of its feet in contact with the ground. The speed was raised from 160 mm·s⁻¹ to 270 mm·s⁻¹, while the characteristics of the gait cycle were steady. Additionally, the torque at the ankle was less than that of the unjointed foot due to the higher height of inexpensive impediments (like stairs). However, there have been proposals for passive toe joints [1, 6]. To sustain the motion and enhance energy consumption, they depend on the employment of spring-damper systems. It has been shown that compliant limbs adapt to challenging terrains more effectively [9]. The foot of the robot WABIAN-2R [6] was modified to replicate the longitudinal arch of a human foot.

a)



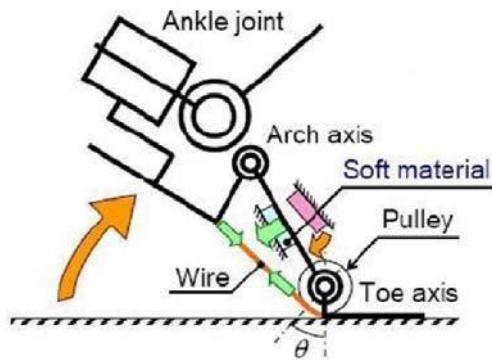


Fig. 1. Robotic feet. a) shows the actuated ankle of the H7 robot [12]; while b) presents an schematic diagram of WABIAN-2R's compliant foot [6]

(see Figure 1b. In addition to having two toes, WABIAN-2R's foot is separated along its sagittal plane, forming a transversal arch. Its soft, elastic material restricts the toe joint's range of motion, and its steel wire replicates the plantar aponeurosis ligament's function. As shown in Fig. 1b, this wire is extended from the heel to the toes. The wire's pull force raises the push-off forces and provides the stride a more organic appearance. The foot suggested by Borovac and Slavnic [1] and Nishiwaki et al. [11] include force sensors in the sole for real-time ZMP computation in order to apply appropriate control techniques. Although the control approach used in WABIAN-2R is not mentioned by Hashimoto et al. [6], the efficacy of their foot in mimicking a human foot was confirmed by comparing ground response forces during push-off and total foot motion.

Model on a bicycle An eight element connection is what we refer to as a walker (see Fig. 2). The joints at the ankles and between the hips and thighs are seen as spherical, while the knees are represented by revolute joints. This permits mobility in a direction normal to the motion direction as well as along the motion direction. A spring-damper mechanism positioned perpendicular to the ground makes up the foot. Figure 2 shows the spring-damper. For the purpose of readability, the mechanism is just shown as a spring. Toe joints are not taken into account. The top portion of the body is seen as a single link with equal mass since we will be concentrating on the lower limbs; this is frequently called HAT (Head and Trunk). With the z-axis normal to the ground surface pointing upward and the y-axis following the motion direction, a global reference frame was established. Table 1 [8] lists the criteria that were taken into account. It is crucial to remember that the moment of inertia (J_i) values are shown using the local frame located at the corresponding segment's center of mass. Since their definition is ambiguous for the approach used here, local frames that are connected to the model's linkages are not shown. We assume the individual to be inside the 5 percentile of male U.S. crew members for analytical purposes.

Point masses that are situated at a distance measured from the link's closest hip joint will be used to depict the limbs.

Table 1. Geometric and dynamic parameters for the 5% U.S. male crew member. [8]

	Shin	Thigh	Trunk
Length [m]	0.47	0.43	0.80
Mass [kg]	3.30	10.60	47.20
J^x [kgm ²]	4.37e-2	12.25e-2	21.53e-2
J^y [kgm ²]	4.30e-2	11.63e-2	25.56e-2
J^z [kgm ²]	0.51e-2	3.16e-2	107.31e-2

1.1. Stability criteria

1.1.1. Zero Moment Point [ZMP]

It is impossible to mention ZMP without referring to M. Vukobratović's works, such as [17, 18], and their importance in the control of biped walking robots. ZMP is defined as the point on the ground where the tipping moment due to gravitational, inertial and reaction forces acting on the biped equals zero. For case of the planar walker, the ZMP coordinates can be determined from equations (1).

Where:

x_{ZMP}, y_{ZMP} – position of the ZMP along the corresponding axis;

m_i – mass of link i ;

x_i, y_i, z_i – position of center of mass of link i along appropriate axis;

g – acceleration due to gravity ($g = -9.81 \text{ m} \cdot \text{s}^{-2}$);

I^j – moment of inertia of link i along axis j expressed on the global reference frame;

ω^j – angular velocity of link i along axis j . These use the notation from [19] (for further detail, interested readers may see [cite{Sardain}, cite{Vukobratovic2004}]) and are valid for both single and double support phases. The coordinate along the z -axis is in the ground plane by definition.

1.1.2. Center of Pressure [CoP]

A point on the ground surface is called CoP. It is positioned such that the total moments brought on by ground reaction forces equal zero. Stated differently, ZMP is determined by the walker's kinematic behavior, while CoP is defined as a function of the recorded ground reaction forces. Despite having different definitions, it can be shown that these two points are really equal [14]. They are hence fairly interchangeable. CoP position may be obtained using (2), which are comparable to those proposed by Schepers et al. [15] and are adapted from the work of Sardain and Bessonnet [14].

$$M = OR \times F_r + OL \times F_l \quad \text{CoP} = \frac{n \times M}{n \cdot (F_r + F_l)}$$

Where:

F_i – pressure force on the corresponding foot (l for left and r for right);

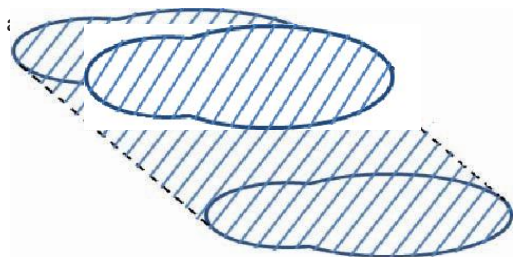
n – vector normal to the ground surface (for as model defined as in Fig. 2, $n = [001]^T$)

OR, OL – distance from the frame's origin to the application point of the corresponding pressure force (see Fig. 2).

Relation (2) will hold for both single and double support phases; and with an appropriate choice of vector n it does not restrict the ground to a horizontal surface.

1.1.3. Usage of stability criteria

For the gait to be deemed stable when using a ZMP/COP based control, the corresponding point must remain within the supporting polygon [3, 17]. This support polygon is constrained by the footprint during the single support phase (see Fig. 3a). It is possible to think of the double support phase as spanning the space between both feet (see Fig. 3b).



It is helpful to know the polygon's form while creating a ZMP/CoP-based gait. In order to do this, Erbatur et al. [3] specify the ideal position of the ZMP point inside this polygon and calculate the leg motion.

2. Reconstruction of the gait and equilibrium conditions.

A VICON motion capture device was used to record a human subject while they walked. Following processing, the angles between the hip, knee, and shin segments are recorded and

used for motion analysis. Below is a thorough description of the procedures that were employed.

2.1. Kinematics

We must determine the biped's position at each moment in order to rebuild the person's gait.

We consider the structure of the biped to be a serial linkage fixed at the hip for every set of observed angles. This will enable us to ascertain where the ankles, knees, and trunk are in relation to the hip frame. The supporting foot or feet are then appropriately positioned in the global reference frame to simulate motion. Joint locations are computed and recorded at each moment in time. The model's location must be offset for each time instant in order to make the supporting foot look stationary. This offset may be utilized to provide a starting condition for the walk and is represented by the vector d_{shift} (see Fig. 4).

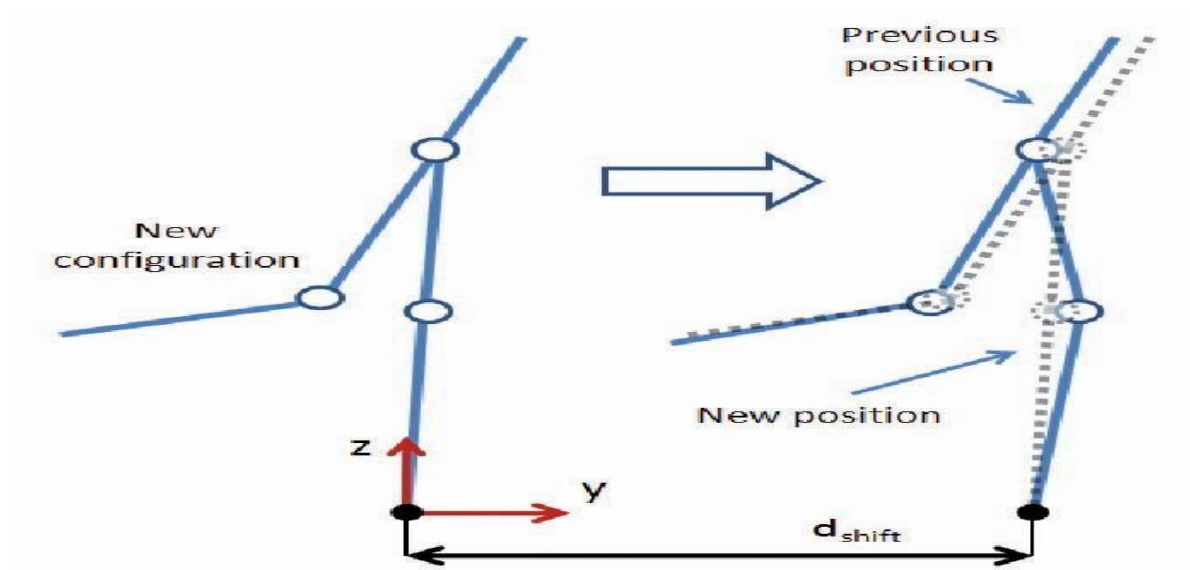


Fig.4. Shifting of the biped to give the impression of motion

Equation (3) may be used to determine the walker's center of mass (PCM) position after the trajectory of each body segment has been determined.

Where:

$$P_{CM} = \frac{\sum m_i P_{CMi}}{\sum m_i}$$

b) Fig.3. Support polygon for a) single support and

b) double support phase P_{CMi} corresponds to the global position link's center of mass;

P_{CM} is the position of the biped's center of mass.

We must first determine the angular velocity and acceleration of the biped's links, as well as the linear acceleration of their centers of mass, in order to get the ZMP of the biped as specified by (1). These values may be found using numerical differentiation. There is no problem in determining response forces under single support. Conversely, we discover that the system lacks a unique solution during the double support phase. To get one, we suppose that a shift function [7,16] represented by f transfers the forces encountered between the supporting legs. The time instants for the start and finish of the single support phases must be known in order to do this. For the single support phase, a function f_i is generated with a value of one; during transfer, it has a value of zero. It is assumed that the transition between the two states occurs linearly throughout the double support phase. Various shift functions have been put forward. Ruiz Garate [16] cites Lei Ren et al. [13]'s work, which uses an exponential function to analyze 3D data with excellent sagittal plane findings. In this case, a linear function (4) produces decent results while being easy to apply.

$$\begin{aligned} \text{during swing} & \quad t_{pi} < t \leq t_{hi} \\ \text{after heel - strike of } i & \quad t_{hi} < t \leq t_{pi} \\ \text{double support} & \quad t_{pi} < t \leq t_{hi} \end{aligned}$$

2.2. Ground reaction force's point of application

CoP is the point at which the total of the moments brought about by the vectors of the ground reaction force equals zero. In light of this, it is essential to identify the location along the foot's sole where the reaction forces are applied (here, represented by the letter "C").

Both the left and right legs have specified functions. Throughout the double support period, this function fluctuates between 0 and 1. Here t_h and t_p refer to the time instant of heel-strike and push-off for the corresponding leg; subindex i refers to the first supporting leg, j to the other one during one complete gait cycle (see Fig. 5).

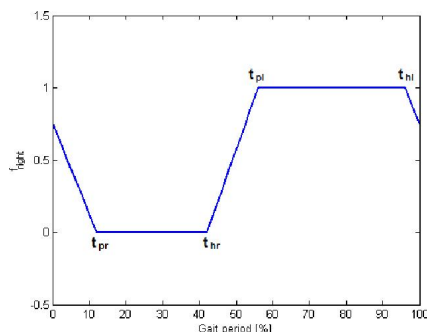


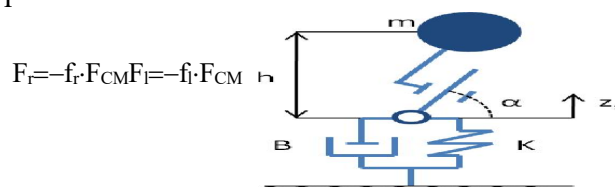
Fig.5. Shift function defined for transfer of ground reaction forces during walk

Following this, ground reaction forces on each leg are given by relation (5).

$$F_{az}$$

2.3. Compliant foot model

We suggest using a spring-damper system situated between the ankle and the sole to simulate the compliance of the foot (see Fig. 6). We assume that deformation will occur only along the z -axis and that this segment is always perpendicular to the ground. The motion of the walker may then be represented as Fig. 7 illustrates. The ordinary differential equation must be solved.



3. Results

We compare the location of point "C," or the application point of ground reaction forces, in the reference frame of the supporting foot (with its origin beneath the ankle, as shown in Fig. 6) after simulating the gait again. Point "C" in the local frame of the supporting foot is shown in Fig. 8; the dotted line denotes a single support, while the solid line denotes a double support phase. When deformation is prohibited, we see big

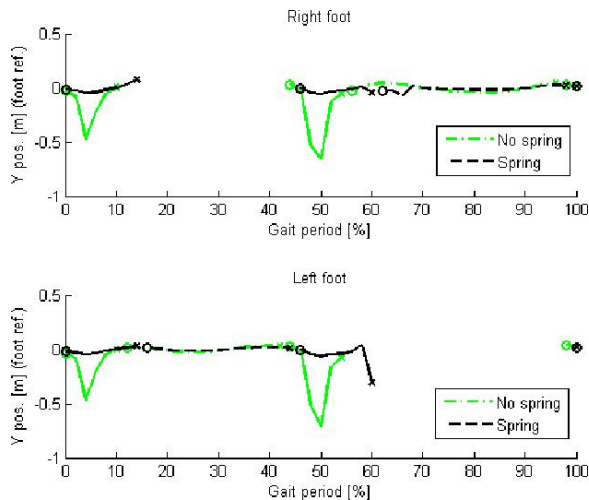


Fig.8.Location of point, "C" expressed in the supporting foot's frame situated outside the foot's sole. Naturally, this is undesirable. The moments brought on by the stresses at the ankle are reduced and "C" stays closer to the ankle after allowing the foot structure to deform (see Fig. 8). The fact that significant vertical accelerations of the biped's center of mass (\ddot{q}) are CM detected at these time instants is noteworthy, even if it is not shown here. According to the conventional inverted pendulum model for biped walking stride, this results from the center of mass's altered trajectory. The CoP point's travel is shown on the ground plane in Fig. 9. We utilize the CoP as determined for a single point contact beneath the ankle for the matching foot (shown with a solid line) as a reference for both the non-compliant and compliant foot (Fig. 9a and Fig. 9b, respectively). Using an un-deformable foot structure, the CoP point changes suddenly during the double support phase. The peaks seen at point "C" are in line with this motion. We see a calmer trajectory after modeling the foot as a compliant element; this is more in line with the straight line, which is what would be predicted given the presumptive conditions. By examining "C's" behavior and how it affects the CoP criterion, we can observe that, similar to "C" (see Fig. 8), CoP moves to the foot's heel before switching the support leg.

4. Future work

Future research should concentrate on the following areas: i) analyzing the evolution of CoP for various motion scenarios; ii) comparing the CoP as determined by direct force measurement to test the suggested method; and iii) examining the impact of toe joints and various spring-damper parameters, each of whose actuation and values are connected to the intended motion and environment. Better estimates of ground reaction forces, such those found by Ruiz Gárate [16] and Geyer et al. [5], could result from these studies. For the suggested model, they may additionally reinterpret how long the single and double support phases last.

3. Conclusions

For the single and double support phases, Vukobratović's stability assessment criteria [17, 18] were presented and assessed. As can be seen from Figs. 3 and 9, a compliant foot structure is desirable for the reconstructions shown here because it maintains the ZMP/CoP within the support polygon. A spring-damper system with viscous friction is suggested as a paradigm for such a compliant construction. The compliancy felt on the foot may also be considered to be a part of the ground because of the features of the biped model that is being employed here.

References

- [1] B. Borovac, S. Slavnic, "Design of multisegment humanoid robot foot", Research and Education in Robotics EUROBOT 2008, vol. 33, Communications in Computer and Information Science.
- [2] S. H. Collins, A. D. Kuo, "Recycling energy to restore impaired ankle function during human walking", PLoS ONE, 5(2):e9307, 02/2010.
- [3] K. Erbatur, Ö. Koca, E. Taşkıran et al., "ZMP based reference generation for biped walking robots", World Academy of Science and Technology, no. 58, 2009, pp. 943–950.
- [4] D. P. Ferris, M. Louie, C. T. Farley, "Running in the real world: adjusting leg stiffness for different surfaces", Proceedings of The Royal Society, 1998, pp. 989–994.
- [5] H. Geyer, A. Seyfarth, and R. Blickhan, "Compliant leg behavior explains basic dynamics of walking and running", Proceedings of The Royal Society, 2006.
- [6] K. Hashimoto, Y. Takezaki, K. Hattori et al., "A study of function of foot's medial longitudinal arch using biped humanoid robot". In: 2010 IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS), Oct. 2010, pp. 2206–2211.
- [7] B. Koopman, H. Grootenboer, H. de Young, "An inverse dynamics model for the analysis, reconstruction and prediction of bipedal walking", Journal of Biomechanics, vol. 28, no. 11, pp. 1369–1376.
- [8] J. T. McConville, T. D. Churchill, I. Kaleps et al., "Anthropometric relationships of body and body segment moments of inertia". In: Report No. AFAM-RL-TR-80-119, Wright-Patterson Air Force Base, OH, 1980.
- [9] L. A. Miller, D. S. Childress, "Analysis of a vertical compliance prosthetic foot", Rehabilitation Research and Development, no. 34, 1997, pp. 54–57.
- [10] N. Milne, The ankle and foot in locomotion. 2010. Course material at UWA. Website: <http://www.lab.anhb.uwa.edu.au/hfa213/week5/lec5afoot.pdf>
- [11] K. Nishiwaki, Y. Murakami, S. Kagami et al., "A Six-axis Force Sensor with Parallel Support Mechanism to Measure the Ground Reaction Force of Humanoid Robot". In: Proceedings of IEEE International Conference on Robotics and Automation. ICRA 2002, vol. 3, pp. 2277–2282.
- [12] K. Nishiwaki, S. Kagami, Y. Kuniyoshi et al., "Toe joints that enhance bipedal and full body motion of humanoid robots", Proceedings of IEEE International Conference on Robotics and Automation. ICRA 2002, vol. 3, pp. 3105–3110.
- [13] L. Ren, R. K. Jones, D. Howard, "Whole body inverse dynamics over a complete gait cycle based only on measured kinematics", Journal of Biomechanics, 41, 2008, pp. 2750–2759.
- [14] P. Sardain, G. Bessonnet, "Forces acting on a biped robot. Center of Pressure Zero Moment Point", IEEE Transactions on Systems, Man and Cybernetics, Part A: Systems and Humans, vol. 34, no. 5, Sep. 2004, pp. 630–637.
- [15] H. M. Schepers, E. van Asseldonk, J. H. Buurke, P. H. Veltink, "Ambulatory Estimation of Center of Mass Displacement During Walking", IEEE Transactions on Biomedical Engineering, vol. 56, no. 4, Apr. 2009, pp. 1189–1195.
- [16] V. Ruiz Gárate, "Inverse Dynamic problem of human gait – Investigation for robotic application", Master diploma thesis, Warsaw University of Technology, 2010.
- [17] M. Vukobratović, "Dynamic models, control synthesis and stability of biped robots gait", CISM Courses and Lectures, no. 375, 1997, pp. 153–190.
- [18] M. Vukobratović, B. Borovac "Zero Moment Point – thirty five years of its life", International Journal of Humanoid Robotics, no. 1, 2004, pp. 157–173.
- [19] T. Zielińska, Postural equilibrium in two-legged locomotion, Working material. Warsaw, Poland, 2010.