International Journal of Applied Electrical and Electronics Engineering
Volume1, Issue 2, 2025

DENOISING WITH SOFT THRESHOLDING

Himanshu, Singh
Department of Engineering, Germany

Research Article

Date of Submission: 20-04-2025 Date of Acceptance: 15-05-2025 Date of Publication: 19-06-2025

Abstract:A comparison of several wavelet denoising methods is presented in this research, and the outcomes were analyzed. Through wavelet domain thresholding, the denoising technique eliminates noise. The "rigrsure" approach is shown to provide the best results. One advantage of the discrete wavelet transform is that it provides a combined time-frequency representation of the signal. Additionally, it is the best system in the area of signal detection and works with both stationary and non-stationary signals. Digital filter banks and multi-resolution analysis are used to achieve the discrete wavelet transform.

Keywords: Wavelet transform, discrete wavelet transform, thresholding , signal denoising

INTRODUCTION

Wavelets are functions used to represent data or other functions that meet certain mathematical conditions. Data is processed by algorithms at various resolutions or sizes. Gross characteristics would be seen if we viewed a signal (or a function) via a huge "window." In a similar vein, we would detect minute details if we looked at a signal via a tiny "window." Wavelet analysis produces a picture of the forest and the trees combined. Because of this, studying wavelets is both fascinating and practical.Many scientific investigations include data sets that are tainted with noise due to environmental factors or the data collection technique. Denoising, or estimating the unknown signal of interest from the existing noisy data, is a preliminary pre-processing step in the analysis of such datasets. Denoising signals and pictures may be done in a number of ways. There are minor distinctions between denoising, de-blurring, smoothing, and restoration even if their visual impacts are identical.

In general, smoothing preserves low frequency while eliminating high frequency (with blurring). While denoising attempts to eliminate all noise present regardless of the spectral richness of a noisy signal, de-blurring enhances the sharpness of signal characteristics by enhancing the high frequencies.Wavelet transformations allow us to express signals with a high degree of sparsity. Restoration is a kind of denoising that attempts to recover the original signal by optimally balancing de-blurring and smoothing. This is the idea underlying wavelet denoising, a non-linear wavelet-based signal estimation method. Regardless of the frequency composition of the signal, wavelet denoising aims to eliminate the noise while maintaining the signal's properties. It is important to distinguish between wavelet denoising and smoothing. As previously stated, smoothing keeps the lower frequencies while eliminating the higher ones.

I. WAVELET BASED DENOISING

Donoho does the majority of the denoising effort, which is based on thresholding the signal's DWT. The technique is based on the observation that noise often appears in the signal as fine-grained structure, and WT offers a scale-based decomposition. At finer scales, the wavelet coefficients tend to reflect the majority of the noise. If these coefficients were eliminated, noise would naturally be filtered out based on scale. The Donoho approach filters the wavelet coefficients to zero if their values are below a threshold since the coefficients at such scale also often serve as the main bearers of edge information. The majority of these coefficients relate to the noise. Conversely, the signal's edge-related coefficients are often higher than the threshold.

Soft thresholding is an alternative to harsh thresholding that results in less severe signal distortion. A number of methods have been proposed for determining the threshold for each wavelet decomposition band. Calculating the sample variance of the coefficients in a band and setting the threshold to a multiple of the deviation is a popular method.

There are several uses for wavelet denoising in signal processing and other domains. There are three possible dimensions for the signals: one, two, and three. They include important information. In many

applications, the initial step is denoising (noise reduction). Additional uses include radio astronomy image analysis, data mining, and medical signal/image analysis (ECG, CT, etc.).Motivation to the thresholding idea is based on the assumptions that the decorrelating property of a wavelet transform creates a sparse signal: most untouched coefficients are zero or close to zero. Also noise is spread out equally

along all coefficients.The noise level is not too high so that we can distinguish the signal wavelet coefficients from the noisy ones.

As it turns out, this method is indeed effective and thresholding is a simple and efficient method for noise reduction. Further, inserting zeros creates more sparsity in the wavelet domain and one can see a link between wavelet denoising and compression.

Hard and soft thresholding with threshold are defined as follows: The hard thresholding operator is expressed in equation(1) as,

$$D(U, \lambda) = U \text{ for all } |U| \geq \lambda \quad (1)$$

$$= 0 \text{ otherwise}$$

The soft thresholding operator on the other hand is expressed in equation (2) as,

$$D(U, \lambda) = \text{sgn}(U)(|U| - \lambda) \quad (2)$$

$$\text{if } |U| \geq \lambda$$

Hard threshold is a more intuitively attractive "keep or kill" process. Soft thresholding is an approach that reduces the absolute value of coefficients over the threshold. Hard thresholding may seem natural at first, but there are certain benefits to continuing with soft thresholding. It improves the mathematical tractability of algorithms. Pure noise coefficients may sometimes cross the hard threshold and show up in the output as bothersome "blips." These misleading structures are reduced by soft thresholding.

III THRESHOLDING ALGORITHMS

The threshold selection is a basic problem. Too many coefficients are reduced by a very high threshold, which causes oversmoothing. On the other hand, a threshold value that is too low permits the inclusion of several coefficients in reconstruction, resulting in an estimate that is wavy and under-smoothed. These considerations must be carefully balanced in order to choose the right threshold. Donoho and Johnstone are primarily responsible for the majority of the work. Global thresholding and level-dependent thresholding are the two primary categories into which a variety of threshold selection techniques fall. While the latter selects a separate threshold value j for each wavelet level j, the former selects a single value of to be applied globally to all empirical wavelet coefficients.

A. Universal Thresholding - *'sqtwolog'*

This type of global thresholding method was proposed by Donoho and Johnstone. This is also called „sqtwolog" method. The threshold value is given in equation (3) as

$$\lambda = \hat{\sigma}\sqrt{2\log n} \quad (3)$$

where n is the number of data points, and $\hat{\sigma}$ is an estimate of the noise level σ. Donoho and Johnstone proposed an

estimate of σ that is based only on the empirical wavelet coefficients at the highest resolution level $J-1$ because they

consist most of noise. Most of the function information except the finest details is in lower level coefficients. The median of absolute deviation (MAD) estimator is expressed in equation (4) as

$$\frac{\text{median}(|w_{J-1,k}|)}{\text{median}(w_{J-1,k})} \quad (4)$$

$$)) 0.6475$$

The universal thresholding removes the noise efficiently. The fitted regression curve is often very smooth and hence visually appealing. If $z_1 \ldots z_n$ represent the wavelet coefficients of the noise with idd N $(0, \sigma^2)$, then it is expressed in equation (5) as

$$\lim P(z_i \leq \sqrt{2\log n}) \quad \frac{1}{n}$$

(5)

This means that the probability of all noise being shrunk to zero is very high for large samples. Since the universal thresholding procedure is based on this asymptotic result, it sometimes does not perform well in small sample situations.

B. Minimax Thresholding

Mini max is another global thresholding method developed by Donoho and Johnstone.

Assume that given observations $w_i = \theta_i + z_i, i = 1, \ldots, n$ is according to equation (6) (6)

where $z_i \sim N(0, 1)$, $\varepsilon > 0$. We wish to estimate the l^2 risk

$$R_{l^2}(\hat{\theta}, \theta) = E \left\| (\hat{\theta}, \theta) \right\|^2 \quad (7)$$

Let $\eta_s(\cdot,\cdot)$ be the soft thresholding function defined in equation (4.2). Suppose we have a single observation y~N (μ, 1). The function is defined in equation (8) as

$$\eta_s(\theta,\lambda) = E_\theta(y-\theta)^2 \quad (8)$$

and the minimax quantities are expressed in equation (9) as,

$$\sup_\theta \inf_\lambda \frac{\eta_s(\theta,\lambda)}{\frac{1}{n}+\min(\theta^2,1)} \quad (9)$$

λ_n^* = the largest λ attaining η_n^* above.

Theorem 1: Assume model (6) and (7) are given. The minimax threshold λ_n^* defined by (9) yields an estimator $\hat{\theta}_i^*(y_i)\ i=1,\ldots,n$ which satisfies $E_\theta\|\hat{\theta}^*-\theta\|^2 \le n\left(\frac{1}{n}+\min(\theta_i^2,1)\right)$ for all $\theta\in R^n$,

where $\lambda_n^* \sim \sqrt{2\log n}$ and $\lim_{n\to\infty}\frac{\lambda_n^*}{\sqrt{2\log n}}=1$ (10)

$\eta_n^*(\lambda_n^*)\sim\frac{1}{\sqrt{2\log n}}$ and $\lim_{n\to\infty}\frac{\eta_n^*(\lambda_n^*)}{\frac{1}{\sqrt{2\log n}}}=1$ (11)

Compared with universal threshold, the minimax thresholding is more conservative and is more proper when small details of function f lie in the noise range.

C. Sure Shrink – *'rigrsure'*

Sure Shrink chooses a threshold λ_j by minimizing the Stein Unbiased Risk Estimate for each wavelet level j . It is also considered as „rigrsure" method.

Theorem4.2: Let $\theta=(\theta_i : i=1,\ldots,d)$ and $x=(x_i : i=1,\ldots,d)$ where $x_i\sim N(\theta_i,1)$. Let $\hat{\theta}=\hat{\theta}(x)$ be a fixed estimator of θ.

The l^2 loss $\|\hat{\theta}-\theta\|_{l^2}^2$ can be estimated unbiasedly for an estimator of θ in the form of

$$\hat{\theta}(x)=x+g(x) \quad (12)$$

where the function $g=(g_1,\ldots,g_d)$ from R^d to R^d is weakly differentiable.

Then the risk given by

$$E_\mu\|\hat{\theta}-\theta\|^2 = d+E\|g(x)\|^2+2\sum_{i=1}^{d}\frac{\partial}{\partial x_i}g_i(x) \quad (13)$$

where $\frac{\partial}{\partial x}g(x)=\left(\frac{\partial}{\partial x_i}g_i(x)\right)_{i=1}^{d}$

We use the soft thresholding function

$$g(x) = \begin{cases} x - t, & x_i > t \\ x + t, & x_i < -t \\ 0, & |x_i| \le t \end{cases} \quad (14)$$

Hence

$$\min_{t} \| g(x) \|_2^2 = \sum_{i=1}^{d} \min(x_i^2, t^2) \quad \text{and} \quad \sum_{i=1}^{d} \min(x_i^2, t^2) \le t^2 \quad (15)$$

Under this case, by (4.13), we have

$$SURE(t; x) = d - 2 \cdot \#\{i : |x_i| \le t\} + \sum_{i=1}^{d} \min(x_i, t) \quad (16)$$

where $|x| \wedge t = \min(|x|, t)$ and # means "the number of". (16) is an unbiased estimate of risk

$$E\left\| \hat{\mu}(t) - \mu \right\|^2 = E\, SURE(t; x)$$

By use of $E\, SURE(t; x)$ as the estimator of risk, the SURE threshold is given by

$$t^* = \arg\min_{0 \le t \le \sqrt{2 \log d}} SURE(t; x) \quad (17)$$

At each wavelet coefficient level j, we apply the above method with $x = (w_{j,k})_{k=0}^{2^{j-1} d - 2^j}$. Recall $w_{j,k} \sim N(\theta_{j,k}, \sigma^2)$, where $\theta_{j,k}$ is the wavelet coefficient of f which is going to be estimated and σ is the noise

level. In order to apply SureShrink method, we need to rescale $w_{j,k}$ by use of the MAD estimate in (4.4) for σ. The

computational effort involved in minimizing the SURE criterion is light. Donoho show that the whole effort to calculate

$\square_j$ in level j with $d \square 2^j$ wavelet coefficients is $O\square d \log\square d \square\square$.

D. *'heursure' method*

When the wavelet representation at any level is very sparse—that is, when the great majority of coefficients are practically zeros—SureShrink performs poorly. Donoho recommends a combination of SureShrink and universal threshold. The global threshold is employed if the collection of coefficients is sparse; otherwise, SURE is used. This hybrid approach is known as Heursure.

IV. IMPLIMENTATION – WAVELET DENOISING

The performance of four conventional denoising algorithms – namely „rigrsure", „heursure", „sqtwolog' and „ minimax „is compared .The denoising procedure is explained in the flow chart shown in Figure 1.

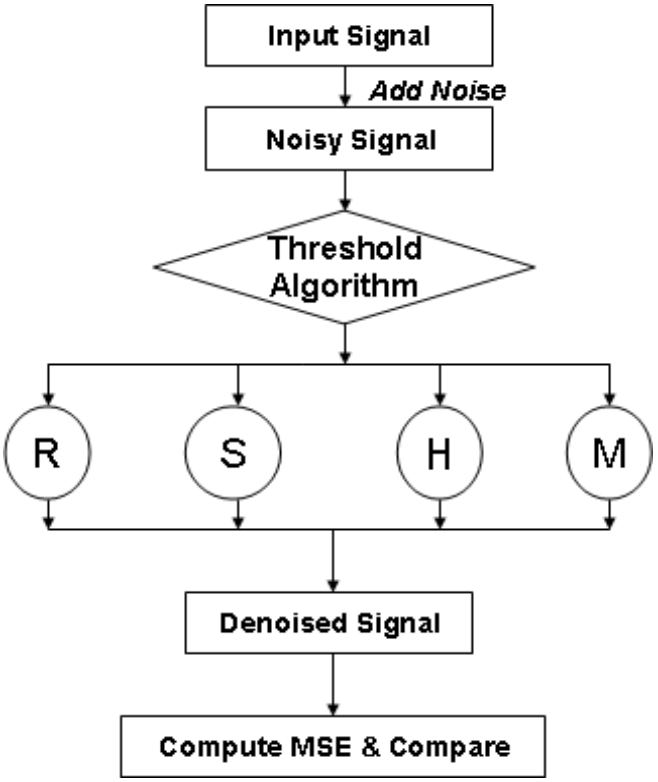


Fig 1 Denoising Procedure

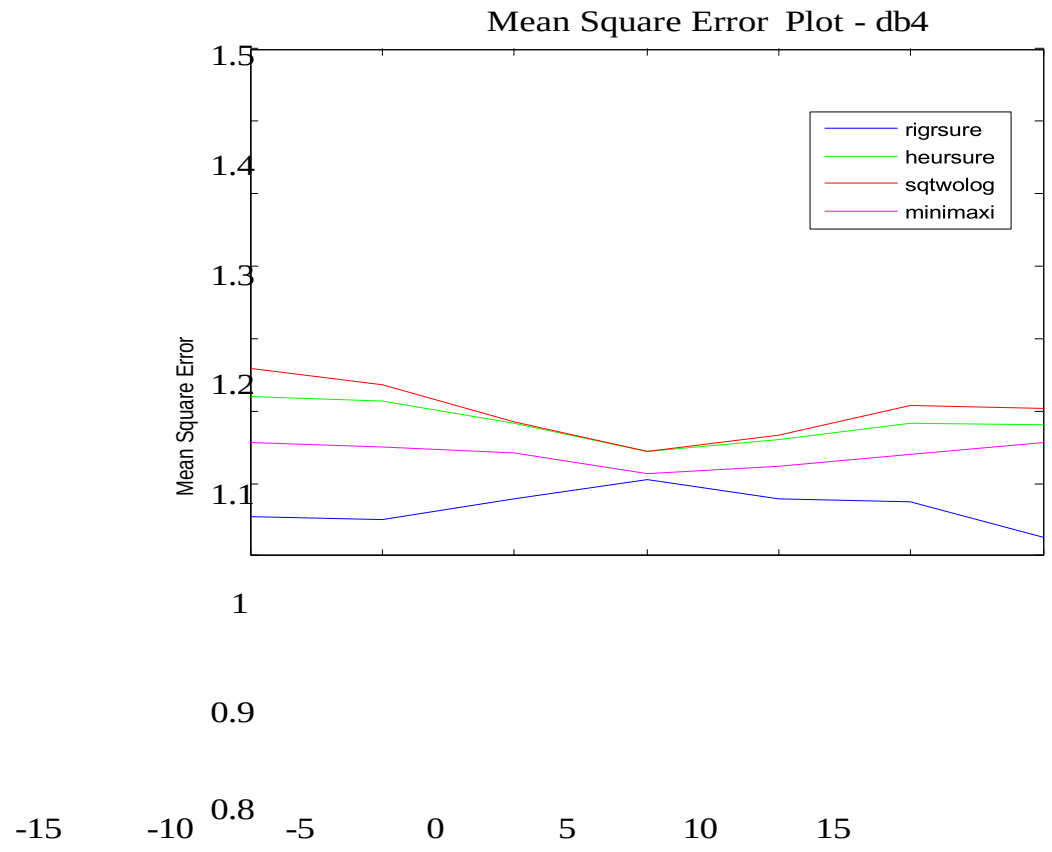
Denoising is accomplished using the four techniques. Soft thresholding is the kind of thresholding that is used. The performance of the denoised signal is compared using the calculated mean square error. The Matlab toolbox, which is often used for high-performance numerical calculation and visualization, is utilized to do this. db4 is the wavelet that is used. Discrete wavelet analysis became feasible when Ingrid Daubechies created what are known as compactly supported orthonormal wavelets. The most often used wavelets are Daubechies wavelets. They serve as the cornerstones of wavelet signal processing and have a wide range of uses. The wavelets of the Daubechies family are named dbN, where db is their "surname" and N is their order.

TABLE 1

PERFORMANCE COMPARISON BASED ON MSE USING DB4

SNR	'rigrsure'	'heursure'	'sqtwolog'	'minimax'
-15	0.8545	1.0195	1.0596	0.9572
-10	0.8498	1.0146	1.0372	0.9500
-5	0.8783	0.9825	0.9847	0.9413
0	0.9052	0.9440	0.9442	0.9134
5	0.878	0.9612	0.9674	0.9229
10	0.874	0.9839	1.0079	0.9410
15	0 .825	0.9814	1.0043	0.9557

The performance measure chosen is the mean squared error (MSE) between denoised signal and the original signal as shown in Table 1.The variables in the set of experiments is different signal to noise ratio ranging from -15 to +15 including zero. If the mean square error is plotted against various SNR values, a graph as shown in figure2 is obtained.



SNR in db

Fig2 Mean Square Error Plot for db4

CONCLUSION

The wavelet technique is appealing due to computational proficiency. Rigrsure yields the best results, according to the set of trials. It has been noted that minimax and heursure perform better than sqtwolog. The kind of signal under consideration and the wavelet used affect denoising performance.

REFERENCES

[1] [1] Agostiino Abbate, Casinemer M. DeCusatis, and Pankaj K. Das, "Wavelets and Sub bands," Boston: Birkhauser, 2002
[2] [2] "Wavelet Transforms – Introduction to Theory and Applications" by Raghuveer M. Rao and Ajit S. Bopadiker, Pearson Education Asia, 1998.
[3] [3] "Insight Into Wavelets -From Theory to Practice," K.P. Soman & K. I. Ramachandran, Prentice Hall, India, 2004
[4] [4] Donoho, D.L. (1993), "Progress in wavelet analysis and WVD: a ten minute tour," in Y. Meyer and S. Roques, "Progress in wavelet analysis and applications," pp. 109–128. Ed in the front.
[5] [5] Donoho, D.L., and I.M. Johnstone (1994), "Ideal spatial adaptation by wavelet shrinkage," Biometrika, vol. 81, pp. 425-455.
[6] [6]Donoho, D.L. "De-noising by soft-thresholding," IEEE Trans. on Inf. Theory, 41, 3, pp. 613-627 (1995)
[7] [7] Mallat, G. A sparse tour of signal processing using wavelets. Academic Press, 2009, third edition [8] www.mathwork.com [9] www.wavelet.org.